Airfoil Flutter:
A Study of Different Models Implementing Steady and Quasi-Steady Aerodynamics

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In aerodynamic design, airfoil flutter is an important consideration because it can be the cause of large amplitude oscillations that could lead to structural failure of the airfoil. The aim of this project was to study several models that aim to predict the behavior of an airfoil at different airspeeds below, at and above the critical airspeed. In particular, the prediction of the critical speed and the ability of the model to accurately predict airfoil plunge and pitch displacements were studied. Through this project, we have concluded that a model implementing steady aerodynamics is able to accurately predict the critical airspeed at which flutter occurs because it is a better model of the steady-state response of the airfoil. However, a model implementing quasi-steady aerodynamics is able to more accurately predict the plunge and pitch behavior of the airfoil becuase it attempts to account for time-dependent effects of the airstream on the airfoil. Despite these conclusions, this project was limited by our ability to accurately determine some of the parameters of our model as well as account for other factors effecting the airfoil. As such, future work should take into account other non-linearities of the system as well as attempt to better determine the parameters of the system such as damping coefficients and the contribution due to dry friction.

1. INTRODUCTION

In aerodynamic design, airfoil flutter is an important consideration because it can be the cause of large amplitude oscillations that could lead to structural failure of the airfoil. For the purposes of this paper, airfoil flutter can be defined as the unstable, self-excited vibration in which the structure extracts energy from the air stream and often results in catastrophic structural failure [1].

In studying airfoil flutter, one of the key factors is a critical airspeed known as flutter speed. At the flutter speed, the airfoil is able to sustain an oscillation after being given some initial displacement or disturbance. In addition, the ranges of airspeeds below and above the flutter speed are also important. When the airspeed is below the flutter speed, the oscillations are damped and the airfoil returns to equilibrium after an initial displacement or disturbance. However, above the flutter speed, the system behaves as though it were negatively damped, which results in oscillations with an amplitude that grows over time unless non-linearities restrict the growth of the oscillations.

In this project, we will study several models that aim to predict behavior of an airfoil at different airspeeds that are below, at or above the critical airspeed or flutter speed. In order to quantify our results, we have chosen to study the plunge and pitch of the airfoil. The plunge is the vertical displacement of the airfoil while the pitch is the angular displacement of the airfoil. Over the course of this paper, we will study models that implement steady as well as quasi-steady aerodynamics and compare these models to physical measurements. We will then discuss the differences between the simulations and the data and why those differences arise.

[1] From *Introduction to Aircraft Aeroelasticity and Loads* by Jan R Wright and Jonathan E Cooper

2. RATIONALE

We chose to do this project because, in a general sense, we wanted to learn more about airfoil flutter and aerodynamic analysis. Specifically, we were interested in studying why the different models for modeling airfoil flutter existed. We also wanted to become more familiar with the differences between steady aerodynamics and quasi-steady aerodynamics and learn about the differences in behavior of each model and what physical phenomena were responsible for the differences we observed.

3. DERIVING THE MODEL

We begin with assuming a free (i.e. no driving force or airspeed), undamped airfoil with 2 degrees of freedom (DOF). In order to model the behavior of the airfoil, the airfoil is mounted to a wertest stand consisting of linear springs that control the plunge and a torsional rod that controls the pitch of the airfoil. The photograph below illustrates this setup:

Fig. 1. Images of the Full Experimental Setup (Left) and the Rig for the Airfoil (Right) - Images obtained from research conducted by Professor Christopher Lee, associate professor of mechanical engineering at Franklin W. Olin College of Engineering

As can be seen in figure 1, there are 5 linear springs oriented vertically and arranged along the length of the plunge mechanism. The plunge mechanism consists of a pair of slides that allows the airfoil to slide up and down along the slides as it displaces vertically. There is also a torsional rod that is not visible in figure 1. This torsional rod is connected to the shaft running through the airfoil and acts as the torsion spring that controls the pitch of the airfoil.

A mathematical model of this test stand set up can then be derived using the following diagram:

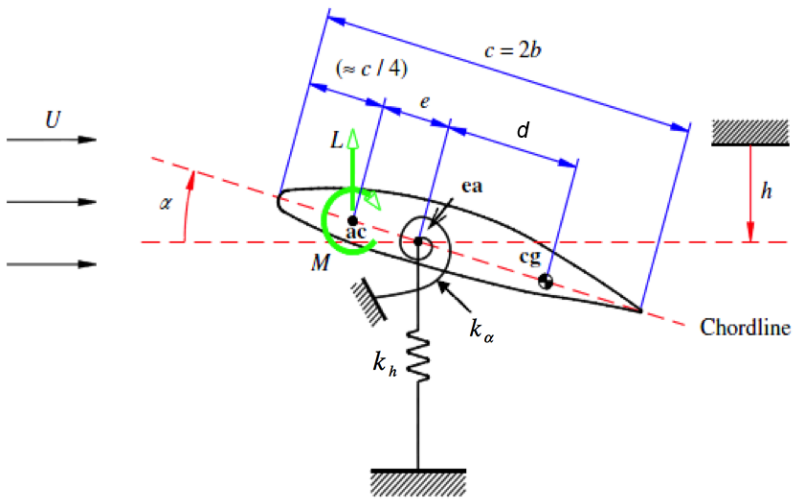

Fig. 2. Diagram of the Airfoil and Definitions of Parameters

The following table lists the system parameters labeled in the above diagram and what they physically represent:

Symbol / Label	Physical Representation
k_h	Spring constant of the linear spring representing the bonding stiffness of the airfoil
k_α	Spring constant of linear torsional spring representing torsional stiffness of wing
$\alpha(t)$	Angle of attack or angular displacement or pitch of the airfoil
$h(t)$	Vertical Displacement or plunge of airfoil
Center of Mass (CM) or Center of Gravity (CG)	The center of mass of the airfoil
Elastic Axis (EA)	Point where the bending and torsion are decoupled
d	The distance between the elastic axis and the center of mass
e	The distance between the Aerodynamic Center (AC) and EA
U	Airspeed
c	Chord Length

3.1 *Equations for a Free, Undamped Airfoil*

The plunge and pitch of the airfoil can be related using the following diagram and equations:

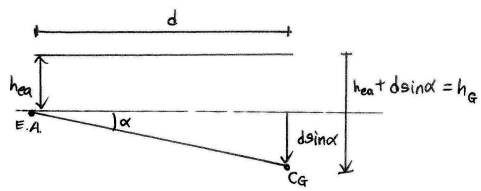

Fig. 3. Diagram Showing Positions of EA, CoM and Aerodynamic Center

$$h_g = h_{ea} + dsin(\alpha) \tag{1}$$

$$\dot{h_g} = \dot{h_{ea}} + d\dot{\alpha}cos(\alpha) \tag{2}$$

$$\ddot{h_g} = \ddot{h_{ea}} + d\ddot{\alpha}cos(\alpha) - d\dot{\alpha}^2 sin(\alpha) \tag{3}$$

Using the sum of linear forces, the following equation is obtained:

$$m(\ddot{h_g}) = m_{tot}\ddot{h_{ea}} + m_{rot}d\ddot{\alpha}cos(\alpha) - m_{rot}d^2 sin(\alpha) + k_h h_{ea} = 0 \tag{4}$$

Where m_{rot} is the total mass of the parts undergoing rotational displacement while m_{tot} is the total mass of the parts undergoing plunge displacement.

The sum of moments can then be considered as follows:

$$\sum \overrightarrow{M_g} = \dot{\overrightarrow{H_g}} \tag{5}$$

$$\sum \overrightarrow{M_{ea}} = I_g \ddot{\alpha_g} \widehat{b_3} + \overrightarrow{r_{g/ea}} \times m_{rot}\overrightarrow{a_g} \tag{6}$$

$$(I_g + m_{rot}d^2 cos^2(\alpha))\ddot{\alpha} + m_{rot}dcos(\alpha)\ddot{h_{ea}} - m_{rot}d^2\dot{\alpha}^2 cos(\alpha)sin(\alpha) + k_\alpha \alpha = 0 \tag{7}$$

3.2 *Adding Viscous Damping*

For this project, we have chosen to implement only viscous damping. This gives rise to inaccuracies in the model compared to the experimental data, which will be discussed later.

Given equations (4) and (7) above for a free, undamped airfoil, viscous damping can be added by adding a term that is proportional to the plunge velocity and pitch velocity. The resulting equations are as follows:

$$m_{tot}\ddot{h_{ea}} + m_{rot}d\ddot{\alpha}cos(\alpha) - m_{rot}d^2 sin(\alpha) + k_h h_{ea} + c_h \dot{h_{ea}} = 0 \tag{8}$$

$$(I_g + m_{rot}d^2cos^2(\alpha))\ddot{\alpha} + m_{rot}dcos(\alpha)\ddot{h_{ea}} - m_{rot}d^2\dot{\alpha}^2cos(\alpha)sin(\alpha) + k_\alpha\alpha + c_\alpha\dot{\alpha} = 0 \tag{9}$$

3.3 *Adding Steady Aerodynamics*

One option to model the effect of the air stream on the airfoil is to model it using steady aerodynamics. This essentially assumes that the effect of the air stream is constant and does not depend on time. Thus, the equations for the lift force, F_l and moment, M, on the airfoil caused by the air stream are as follows:

$$F_l = \frac{1}{2}\rho U_\infty^2 AC_{L\alpha}\alpha \tag{10}$$

$$M = e \bullet F_l = \frac{1}{2}\rho U_\infty^2 AC_{L\alpha}\alpha e \tag{11}$$

These equations can then be added to the equations (8) and (9) that make up the model in the following manner:

$$m_{tot}\ddot{h_{ea}} + m_{rot}d\ddot{\alpha}cos(\alpha) - m_{rot}d^2sin(\alpha) + k_h h_{ea} + c_h\dot{h_{ea}} = -F_l \tag{12}$$

$$(I_g + m_{rot}d^2cos^2(\alpha))\ddot{\alpha} + m_{rot}dcos(\alpha)\ddot{h_{ea}} - m_{rot}d^2\dot{\alpha}^2cos(\alpha)sin(\alpha) + k_\alpha\alpha + c_\alpha\dot{\alpha} = M \tag{13}$$

Thus the final equations of motion that govern the airfoil under steady aerodynamics are as follows:

$$m_{tot}\ddot{h_{ea}} + m_{rot}d\ddot{\alpha}cos(\alpha) - m_{rot}d^2sin(\alpha) + k_h h_{ea} + c_h\dot{h_{ea}} = -\frac{1}{2}\rho U_\infty^2 AC_{L\alpha}\alpha \tag{14}$$

$$(I_g + m_{rot}d^2cos^2(\alpha))\ddot{\alpha} + m_{rot}dcos(\alpha)\ddot{h_{ea}} - m_{rot}d^2\dot{\alpha}^2cos(\alpha)sin(\alpha) + k_\alpha\alpha + c_\alpha\dot{\alpha} = \frac{1}{2}\rho U_\infty^2 AC_{L\alpha}\alpha e \tag{15}$$

These equations can then be solved in MATLAB using the symbolic toolbox to obtain the equations that govern the plunge acceleration, $\ddot{h_{ea}}$, and pitch acceleration, $\ddot{\alpha}$ of the airfoil.

3.4 *Adding Quasi-Steady Aerodynamics*

An alternative to applying steady aerodynamics is to apply quasi-steady aerodynamics. Quasi-steady aerodynamics may be applied as a way to begin to capture the time-dependent effects of the air stream on the airfoil. There are many different implementations of Quasi-Steady. One relatively simple approach is to assume that at any instant of time, the airfoil behaves with the characteristics of the same airfoil moving with constant heave and/or pitch velocities equal to the instantaneous values [2]. This is known as the quasi-steady assumption. Mathematically, this can be done by adding pitch damping terms to the lift and moment equations that are dependent on the instantaneous pitch and plunge velocities of the airfoil[3]:

[2]From *Introduction to Aircraft Aeroelasticity and Loads* by Jan R Wright and Jonathan E Cooper pg 153

[3]Lift and moment equations obtained from *Aeroelasticity Lecture 1: Introduction - Equations of Motion* by G. Dimitriadis at Universite de Liege

$$F_l = \frac{1}{2}\rho U_\infty^2 A C_{L\alpha}(\alpha + \frac{\dot{h_{ea}}}{U_\infty} + (\frac{3}{4}c - x_f)\frac{\dot{\alpha}}{U_\infty}) \tag{16}$$

where x_f is the distance from the leading edge of the airfoil to the elastic axis

$$M = \frac{1}{2}\rho U_\infty^2 A C_{L\alpha} e(\alpha + \frac{\dot{h_{ea}}}{U_\infty} + (\frac{3}{4}c - x_f)\frac{\dot{\alpha}}{U_\infty}) \tag{17}$$

These equations can then be added to the equations of motion for the airfoil in a similar manner as that done in section 2.3 above to produce the following final equations of motion:

$$m_{tot}\ddot{h_{ea}} + m_{rot}d\ddot{\alpha}cos(\alpha) - m_{rot}d^2 sin(\alpha) + k_h h_{ea} + c_h \dot{h_{ea}} = -\frac{1}{2}\rho U_\infty^2 A C_{L\alpha}(\alpha + \frac{\dot{h_{ea}}}{U_\infty} + (\frac{3}{4}c - x_f)\frac{\dot{\alpha}}{U_\infty}) \tag{18}$$

$$(I_g + m_{rot}d^2 cos^2(\alpha))\ddot{\alpha} + m_{rot}dcos(\alpha)\ddot{h_{ea}} - m_{rot}d^2\dot{\alpha}^2 cos(\alpha)sin(\alpha) + k_\alpha\alpha + c_\alpha\dot{\alpha} = \frac{1}{2}\rho U_\infty^2 A C_{L\alpha} e(\alpha + \frac{\dot{h_{ea}}}{U_\infty} + (\frac{3}{4}c - x_f)\frac{\dot{\alpha}}{U_\infty}) \tag{19}$$

As was done in section 2.3 above, equations (18) and (19) can then be solved in MATLAB using the symbolic toolbox to obtain the equations that goven the plunge acceleration, $\ddot{h_{ea}}$, and pitch acceleration, $\ddot{\alpha}$ of the airfoil.

3.5 Including Stall Angle in the Model

Stall angle was also in the models in a implemented in a simplistic manner. In MATLAB, this was done by using an if statement. When the pitch of the airfoil was below the stall angle of 12 degrees, the lift curve slope $c_{L\alpha}$ was set to 2π. When the pitch of the airfoil was above the stall angle of 12 degrees, the lift curve slope $c_{L\alpha}$ was set to -2π. 12 degrees was chosen as the stall angle as this is the generally accepted stall angle for a NACA0012 airfoil.

3.6 Determining System Parameters

The list below describes how the system parameters used to generate the simulations were obtained:

(1) c, d, e and A are all parameters of the airfoil and were obtained by direct measurements of the airfoil
(2) m_{total} and $m_{rotational}$ were obtained from data on the mass of the plunge and pitch mechanism and the airfoil
(3) The moment of inertia, I_g, of the airfoil was obtained from data provided by the solidworks model of the airfoil
(4) The lift curve slope and stall angle was estimated to be 2π according to an approximation of the lift curve for a NACA0012 airfoil, which is the airfoil we have used
(5) k_h and k were obtained from available data on the spring constants for the linear and torsional springs used in the test stand
(6) c_h and c_θ, the plunge and pitch damping coefficients were obtained through a combination of estimation from data available from previous research conducted using the airfoil as well as adjustments made to account for dry friction damping

4. RESULTS

The critical airspeed at which flutter was achieved was studied first. The graphs below show the experimental data, the steady aerodynamics simulation and the quasi-steady aerodynamics simulation when they are below, at and above the critical air speed.

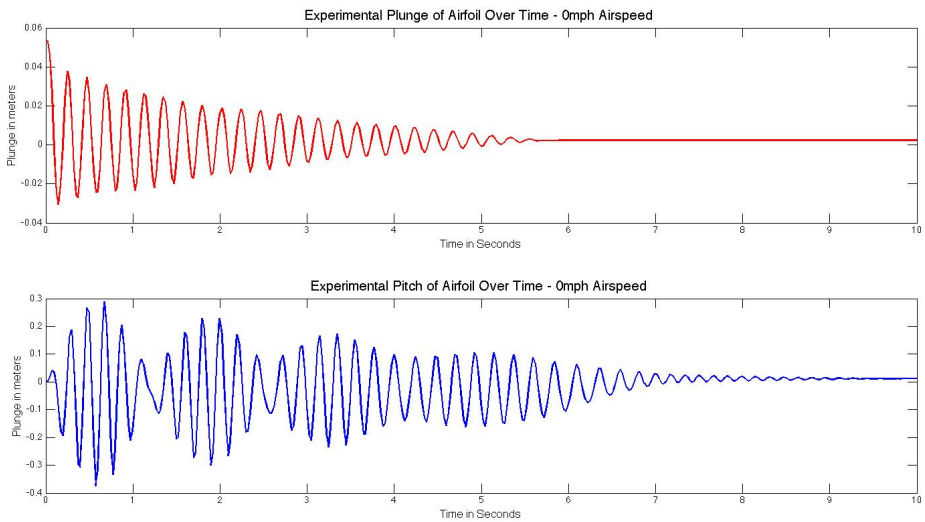

Fig. 4. Experimental Data for the Pitch and Plunge of the Airfoil with a 0mph Airspeed.

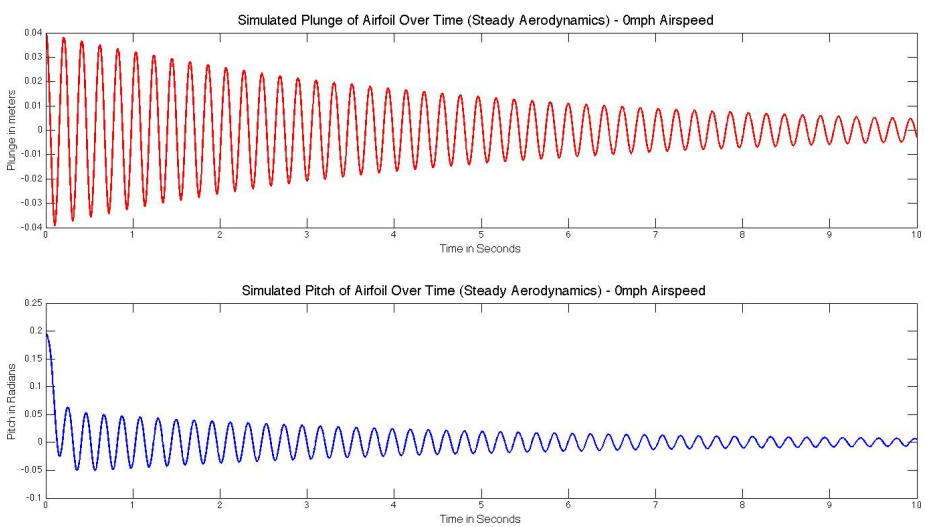

Fig. 5. Steady Aerodynamics Simulation Data for the Pitch and Plunge of the Airfoil with a 0mph Airspeed.

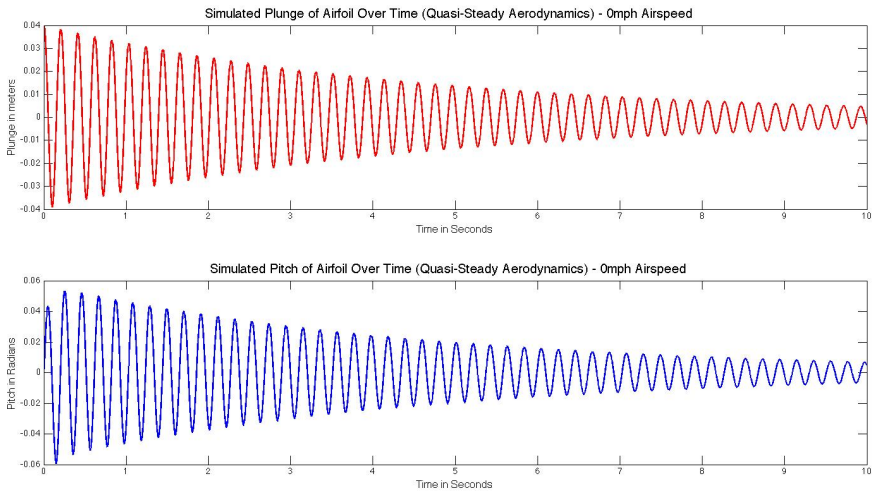

Fig. 6. Quasi-Steady Aerodynamics Simulation Data for the Pitch and Plunge of the Airfoil with a 0mph Airspeed.

By examining figures 4, 5 and 6, it can be seen that the experimental setup as well as the simulation models using steady and quasi-steady aerodynamics are all underdamped when the airspeed is set to zero. Thus, when given an initial plunge displacement, the plunge and pitch oscillations eventually decay away. These graphs prove that the airfoil is at a stable equilibrium in the experimental setup as well as in both simulations.

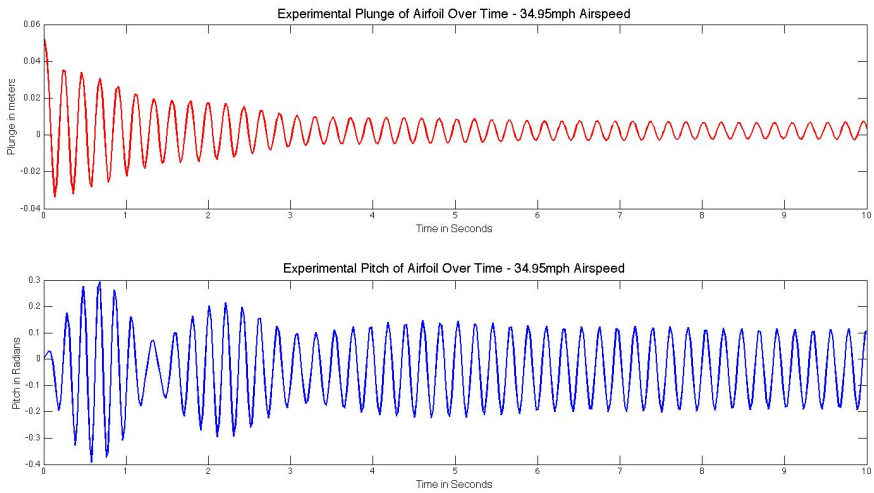

Fig. 7. Experimental Data for the Pitch and Plunge of the Airfoil with a 34.95mph Airspeed.

Figure 7 then shows the experimental setup has reached the critical airspeed required for flutter. This is seen from the steady plunge and pitch oscillations observed in figure 7, where from approximately 4 seconds onward, the pitch and plunge oscillations maintain a constant frequency and amplitude. This behavior is also observed in figure 8:

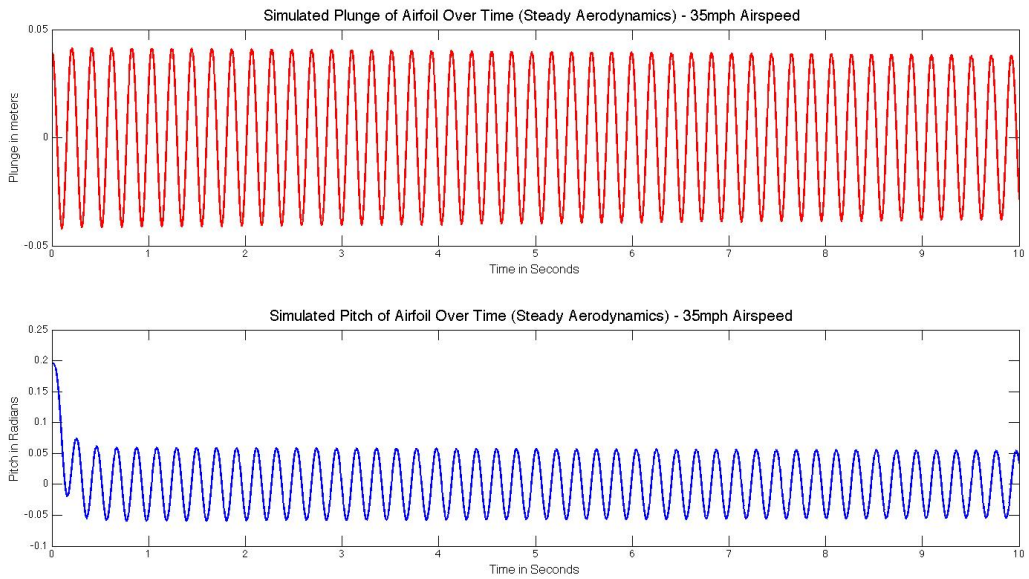

Fig. 8. Steady Aerodynamics Simulation Data for the Pitch and Plunge of the Airfoil with a 35mph Airspeed.

As observed in figure 8, the simulation implementing steady aerodynamics has also reached flutter condition and thus the simulated plunge and pitch of the airfoil remains in constant oscillation over time. The amplitude of the oscillations are, of course, visibly different from the experimental results. This will be discussed later.

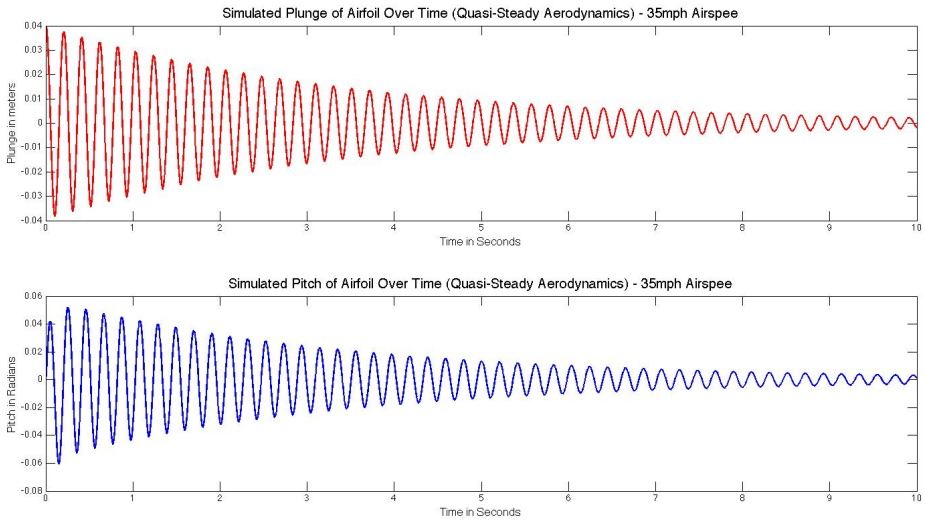

Fig. 9. Quasi-Steady Aerodynamics Simulation Data for the Pitch and Plunge of the Airfoil with a 35mph Airspeed.

On the otherhand, figure 9 shows that the simulation implementing quasi-steady aerodynamics has not reached the critical airspeed yet.

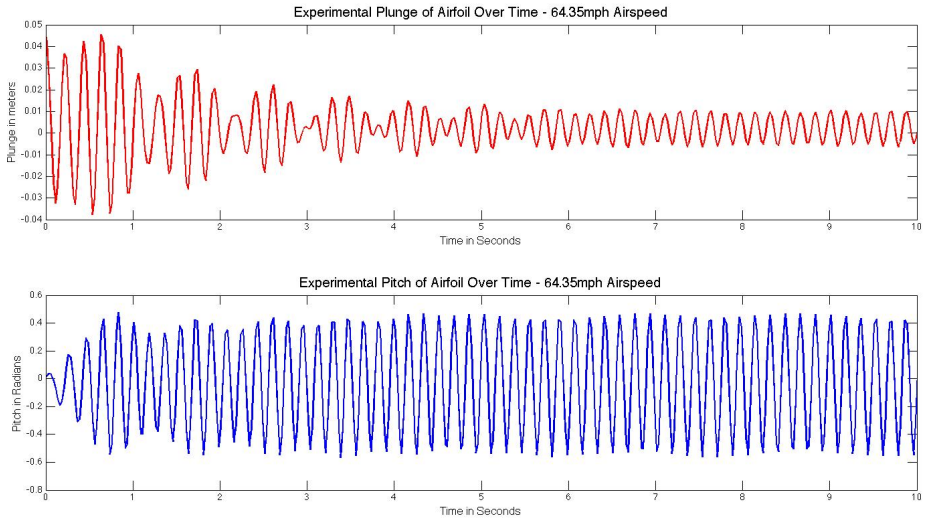

Fig. 10. Experimental Data for the Pitch and Plunge of the Airfoil with a 64.35mph Airspeed.

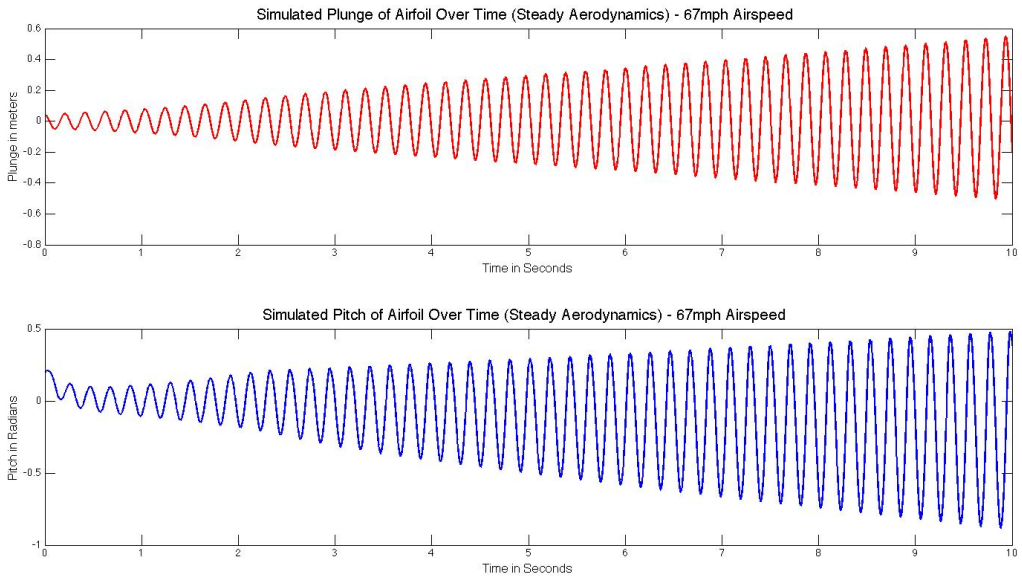

Fig. 11. Steady Aerodynamics Simulation Data for the Pitch and Plunge of the Airfoil with a 67mph Airspeed.

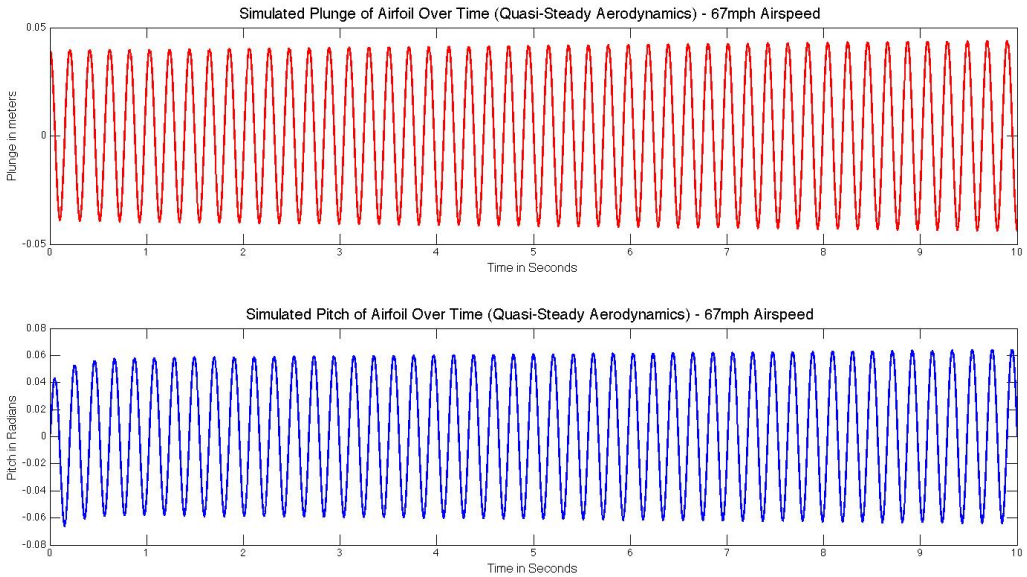

Fig. 12. Quasi-Steady Aerodynamics Simulation Data for the Pitch and Plunge of the Airfoil with a 67mph Airspeed.

The simulation implementing quasi-steady aerodynamics finally reaches critical airspeed with an airspeed of approximately 67mph. As can be seen in figure 10, the experimental setup, when subjected to an approximately 64.35mph airspeed, is experiencing flutter. Figure 11 shows that the simulation implementing steady aerodynamics has exceeded the critical airspeed and thus negative damping is observed. Finally, figure 12 shows that the airfoil has reached critical airspeed and thus the plunge and pitch oscillations maintain a constant amplitude and frequency over time.

Following the above, the accuracy of the models in comparison to the experimental data was studied. The graphs used for that comparison are shown below:

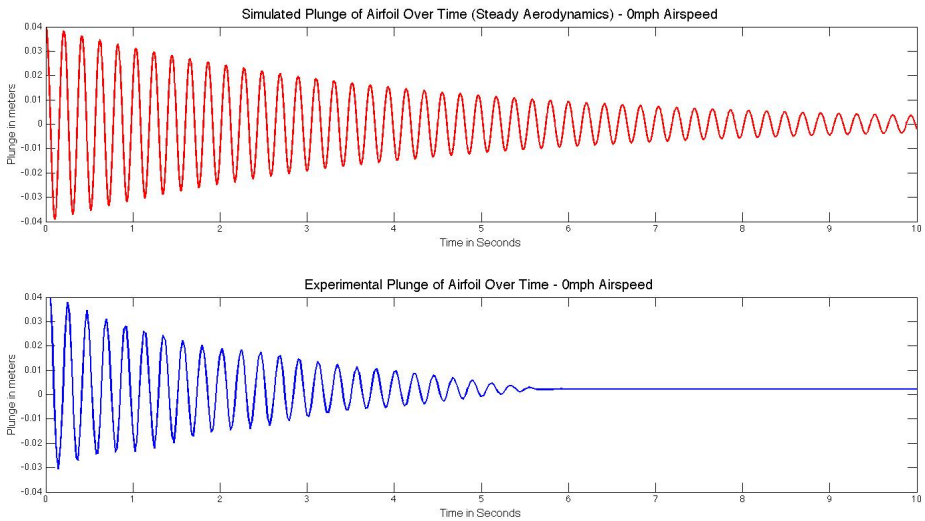

Fig. 13. Plunge of the Airfoil with 0mph Airspeed Using (1) Simulation Data with Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

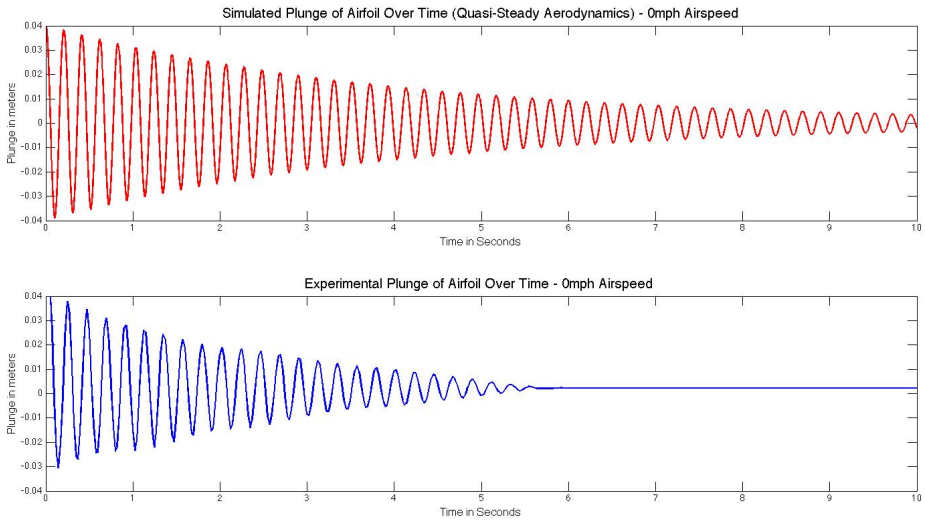

Fig. 14. Plunge of the Airfoil with 0mph Airspeed Using (1) Simulation Data with Quasi-Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

As can be seen from figures 13 and 14, simulations implementing both steady and quasi-steady aerodynamics demonstrate similarly accurate predictions of the plunge behavior of the airfoil compared to the experimental data.

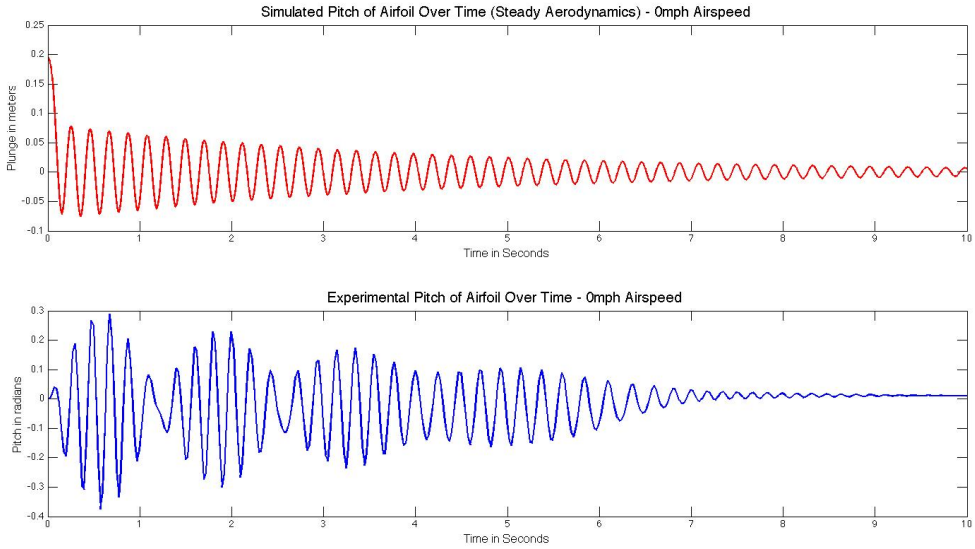

Fig. 15. Pitch of the Airfoil with 0mph Airspeed Using (1) Simulation Data with Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

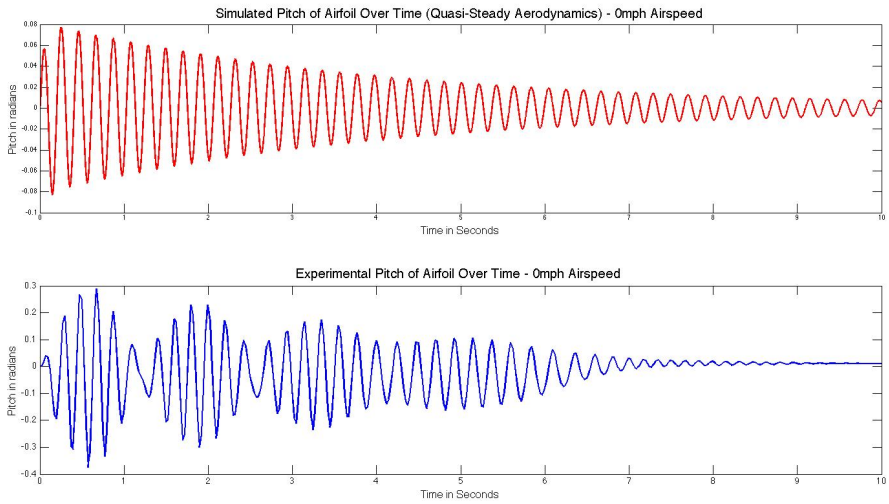

Fig. 16. Pitch of the Airfoil with 0mph Airspeed Using (1) Simulation Data with Quasi-Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

However, figures 15 and 16 show that the simulation implementing quasi-steady aerodynamics produces a more accurate prediction of airfoil pitch behavior compared to the simulation implementing steady aerodynamics. This is seen when the pitch simulation in figure 15 does not match the experimental data closely whereas the pitch simulationi in figure 16 shows a decay envelope that is similar to the experimental data even though the oscillatory behavior does not match precisely.

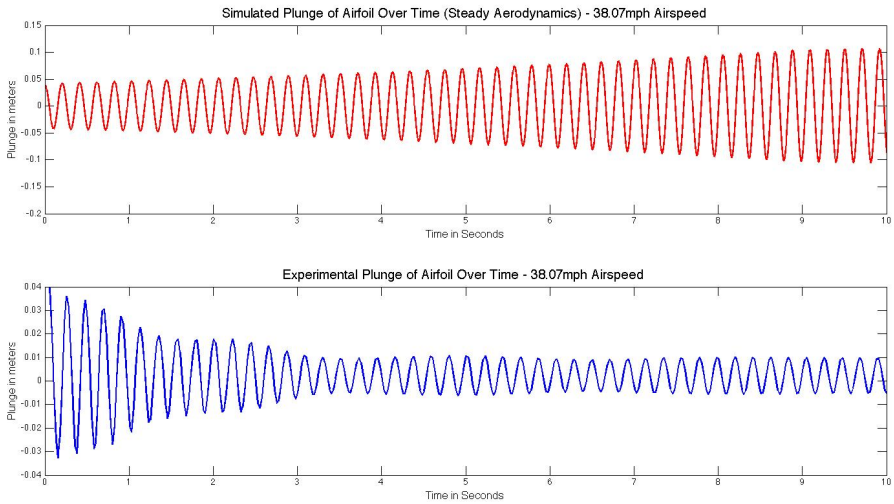

Fig. 17. Plunge of the Airfoil with 38.07mph Airspeed Using (1) Simulation Data with Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

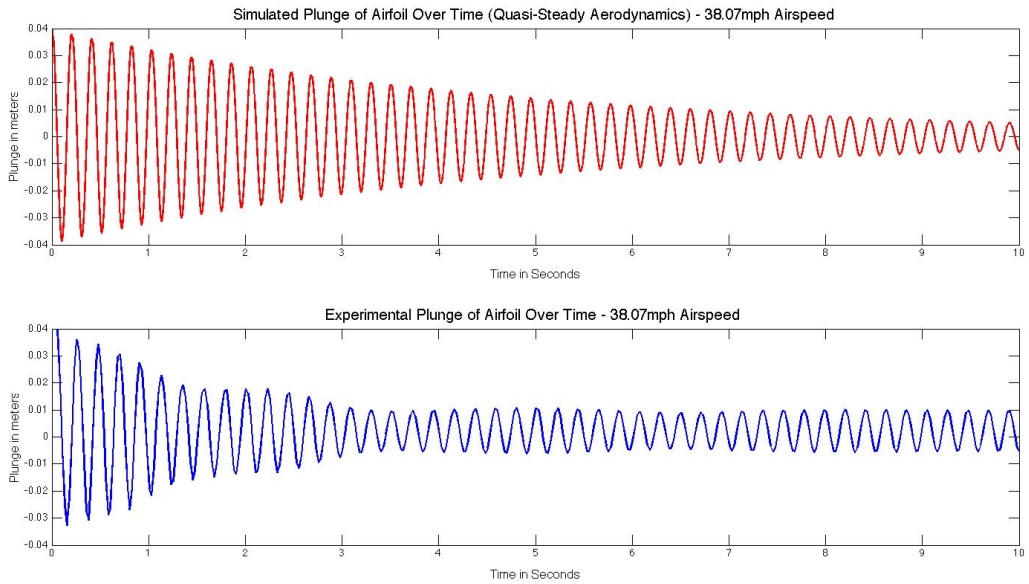

Fig. 18. Plunge of the Airfoil with 38.07mph Airspeed Using (1) Simulation Data with Quasi-Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

Figures 17 and 18 then demonstrate the higher accuracy of the simulation implementing quasi-steady aerodynamics compared to the one implementing steady aerodynamics. As can be seen in figure 17, the simulation implementing steady aerodynamics shows the plunge oscilations diverging or growing whereas in the experimental data, the oscillations decay before settling at a steady state oscillation. In contrast, figure 18 shows the simulation implementing quasi-steady aerodynamics predicts the oscillations to decay at that airspeed in a similar manner to the experimental data.

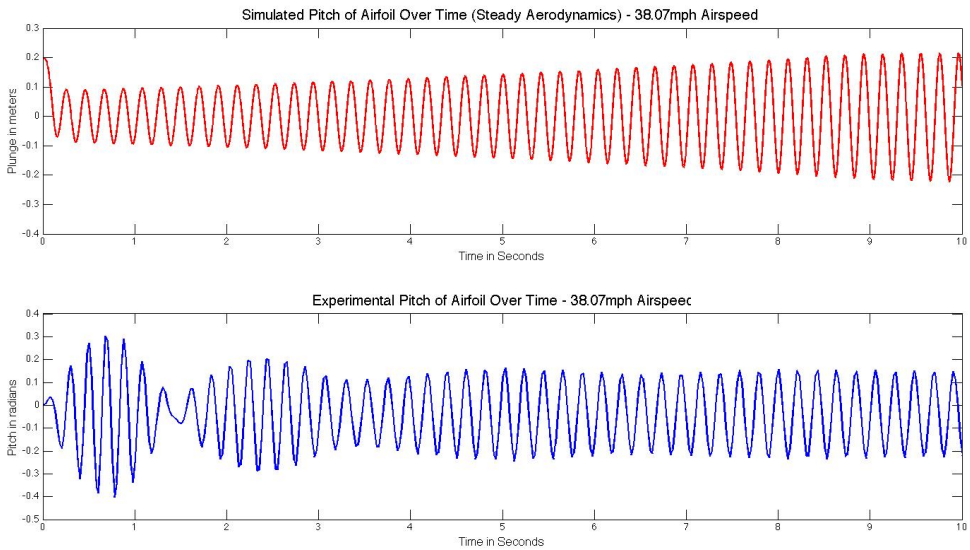

Fig. 19. Pitch of the Airfoil with 38.07mph Airspeed Using (1) Simulation Data with Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

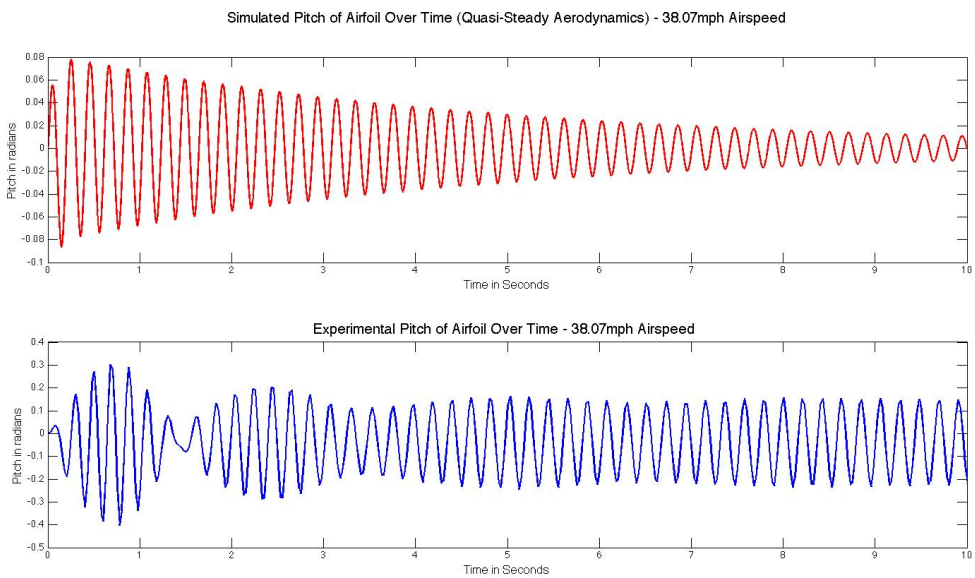

Fig. 20. Pitch of the Airfoil with 38.07mph Airspeed Using (1) Simulation Data with Quasi-Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

Figures 19 and 20 demonstrate the same higher accuracy of the simulation implementing quasi-steady aerodynamics compared to the one implementing steady aerodynamics. As can be seen in figure 19, the simulation implementing steady aerodynamics shows the pitch oscilations diverging whereas in the experimental data, the oscillations maintain a relatively steady frequency and amplitude after approximately 4 seconds. In contrast, figure 20 shows the simulation implementing quasi-steady aerodynamics predicts the oscillations to decay at that airspeed, although the decay is significantly faster than that observed in the experimental data.

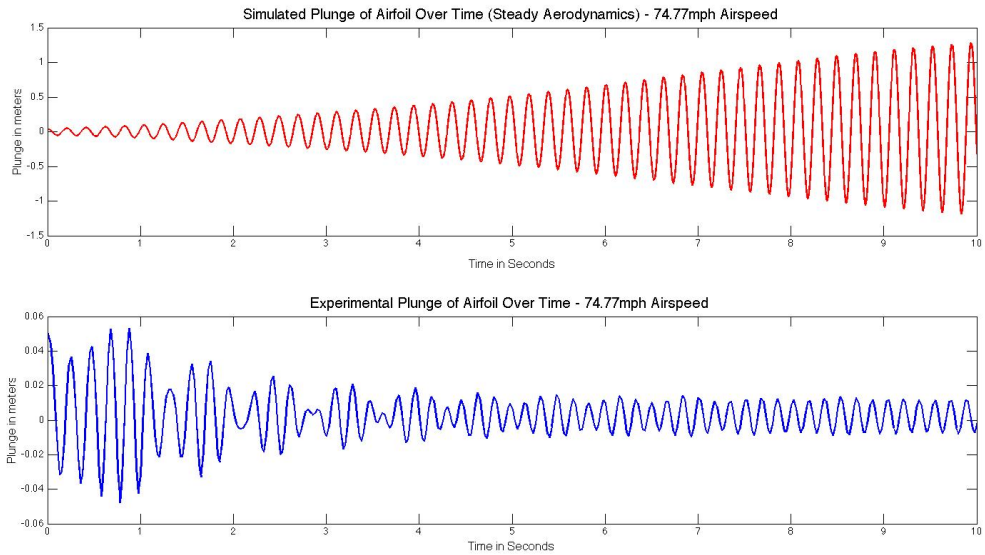

Fig. 21. Plunge of the Airfoil with 74.77mph Airspeed Using (1) Simulation Data with Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

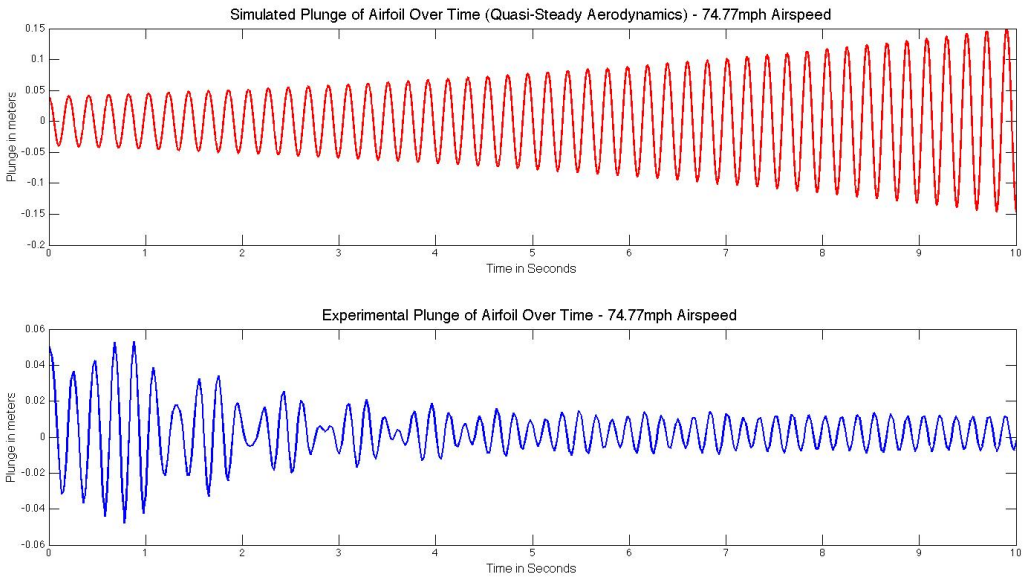

Fig. 22. Plunge of the Airfoil with 74.77mph Airspeed Using (1) Simulation Data with Quasi-Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

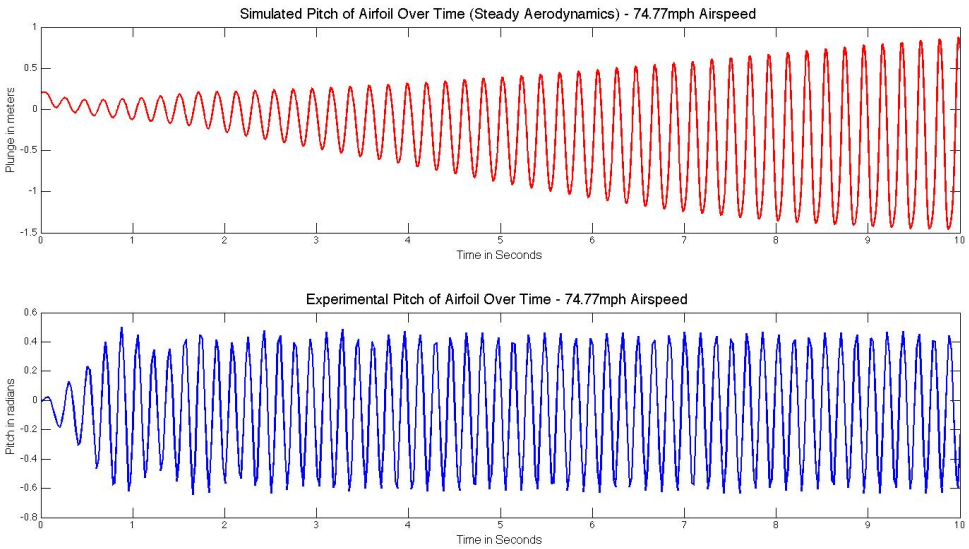

Fig. 23. Pitch of the Airfoil with 74.77mph Airspeed Using (1) Simulation Data with Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

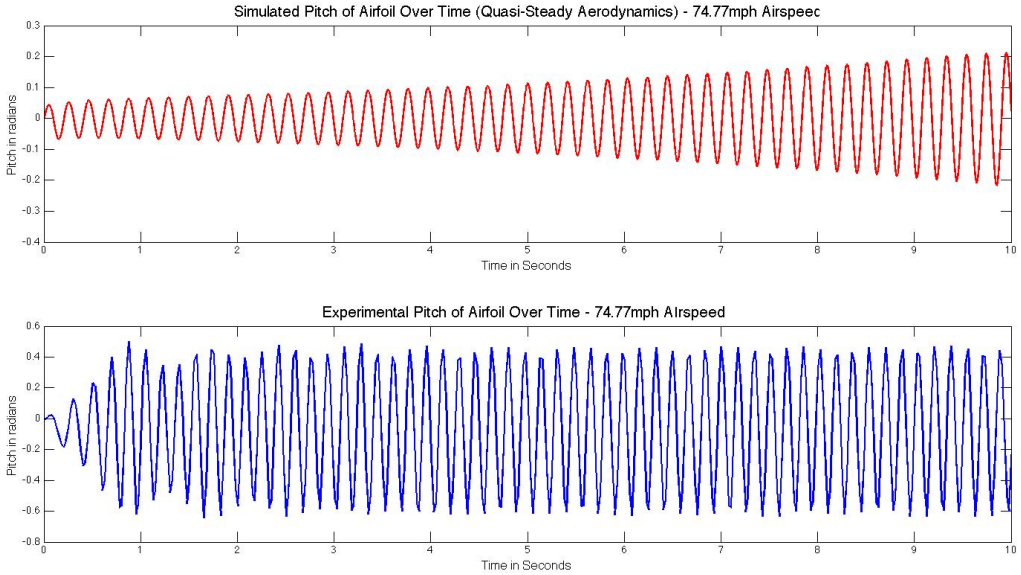

Fig. 24. Pitch of the Airfoil with 74.77mph Airspeed Using (1) Simulation Data with Quasi-Steady Aerodynamics (top) and (2) Experimental Data of the Airfoil (bottom)

As can be seen from figures 21, 22, 23 and 24, neither simulation is able to predict the plunge or pitch of the airfoil with any reasonable accuracy. Although the simulation implementing quasi-steady aerodynamics shows some limit-cycle behavior, the general behavior does not match the experimental data.

5. ANALYSIS AND DISCUSSION

The system behaviors examined here can be split into three cases - airspeeds below the flutter speed, airspeeds at the flutter speed and airspeeds above the flutter speed. As shown above, both the quasi-steady and steady models were simulated at various airspeeds for which experimental data had been collected from the airfoil test set up in the wind tunnel. Comparing the actual data to the simulations of both models showed that neither of the models was overall superior to the other in modeling all three cases. The model implementing steady aerodynamics predicted the flutter speed of the physical system almost exaclty, but demonstrated behaviors that did not accurately reflect the system's plunge and pitch response. The model implementing quasi-steady aerodynamics vastly improved the accuracy of the simulated behavior of the system, although it nearly doubled the predicted flutter speed. Neither model was able to produce viable simulations for speeds above the flutter speed. However the model implementing quasi-steady aerodynamics was able to better demmonstrate the bounding conditions on the growth of the pitch and plunge oscillations due to non-linearities of the system while the model implementing steady aerodynamics was not able to demonstrate these as accurately.

5.1 Prediction of Critical Airspeed

When predicting the critical airspeed of the system, the model implementing steady aerodynamics produced more accurate results than the quasi-steady implementation. The physical system reached critical airspeed at approximately 34.95mph. As can be seen in Figure 8, the steady model predicts the critical airspeed to occur around 35mph which corresponds almost perfectly to the physical system. The quasi-steady model does not demonstrate behaviors indicative of critical airspeed until the airspeed reaches 67mph.

We believe this is the case because one of the key adaptations to the model implementing quasi-steady aerodynamics compared to the model implementing steady aerodynamics was incorporating pitch damping terms that were dependent on the plunge and pitch velocities of the airfoil. By increasing the damping for the quasi-steady implementation, the model became more stable. This increased stability explains why the quasi-steady model does not reach critical airspeed until nearly twice that of the physical system, as the increased stability of the model requires a higher airspeed to cause unstable behavior.

The steady implementation, however, does not include this additional damping and only accounts for the particular solution to the system. As flutter is defined by a sustained oscillation, the point at which an airfoil is considered to have flutter is defined by its steady state response since a sustained oscillation implies that the response of the airfoil is not affected by time-dependent effects. Since this is the assumption made when implementing steady aerodynamics, it naturally follows that the model that does not consider time-dependent effects would accurately predict the critical airspeed required to achive flutter.

5.2 Prediction of Airfoil Behavior

On the otherhand, the quasi-steady implementation of the model demonstrates more accuracy when predicting the actual plunge and pitch behavior of the system than the steady implementation. This can be seen when comparing Figures 15 and 16, 17 and 18, and 19 and 20. While neither model accurately predicts the outer oscillations present in the experimental data in Figures 15-20, the decay envolope of the quasi-steady model more closely matches the experimental data for the physical system as can be seen by comparing the upper and lower graphs in Figures 16, 18 and 20. In comparison, the

decay envolopes for the steady model in Figures 15, 17, and 19 are either not large enough to match the outer oscillations present in the experimental data or diverge and increase while the experimental data continues to decay.

We believe these observations are again due to the way the models are implemented. As discussed earlier, the model implementing steady aerodynamics only accounts for effects on the airfoil that are time independent. However, the quasi-steady implementation attempts to take into account time-dependent effects of the air stream. This suggests that it attempts to capture the transient behavior of the airfoil before it settles into steady-state behavior. Thus, it naturally follows that the model implementing quasi-steady aerodynamics is better able to predict the plunge and pitch behavior of the airfoil. Once the system reaches a state of flutter, the behavior is no longer time dependent. However, before this occurs, or if flutter is not reached, the system behavior changes over time. Incorporating the time-dependencies into the quasi-steady model allows the model to more accurately predict the overall behavior of the system prior to reaching flutter.

Finally, referring to figures 21 through 24, it can be seen that neither implementation is able to accurately predict the behavior of the airfoil. We believe this is because once the flutter speed is exceeded, there are many other non-linearities in the system that are not captured by our model and thus our model is no longer able to accurately predict the experimental data.

6. DIAGNOSIS OF PROBLEMS AND AREAS OF IMPROVEMENT

We experienced several setbacks during this project. One of them was hardware trouble. The computer that was normally used for data collection was out of order. In order to work around the problem, we installed the data acquisition software on one of our computers, which also took significantly longer than anticipated. These hardware troubles set us back a significant amount of time during the initial phase of the project.

We also discovered early into the project that the linear equations of motion were inadequate to match the experimental data we obtained from the wind tunnel. Thus, we had to spend extra time rederiving the equations to use the non-linearized system instead of the linearized system. We also had to adjust all of our simulations to make use of the non-linearized equations because the non-linearized equations could not be written in the same matrix-vector form as the linearized equations of motion.

Thirdly, we discovered fairly late into the project that our model for the damping present in the system was inadequate because dry friction was a relatively dominant factor in the physical system which we modeled using viscous damping. This made our model inaccurate because dry friction is usually modeled with linear damping instead of viscous damping. We initially attempted to implement damping due to dry friction in our simulations but were unsuccessful and thus this remains as future work.

Finally, we used a very basic implementation of stall which simply assumes the lift curve is symmetric and has a linear slope before and after the stall angle. This is, of course, not true of a NACA0012 airfoil, as evidenced by the numerous lift curves for NACA0012 airfoils available in literature and online. However, we did not have the time to implement a more accurate version of stall and thus this also remains as future work.

7. REFLECTION

Through the course of this project, we gained an intuition for the behavior of an air foil in an air stream and the qualitative reasons behind them. Because we spent a long time running simulations and observing the actual behavior in the wind tunnel, we gradually began to get a feel for what to expect from the behavior of the airfoil and the qualitative reasons for why we were observing certain behavior.

This project also taught us a lot about incremental development of a simulation. We initially started with the linearized system of equations and realized that they were insufficient. We then developed a non-linearized model and gradually added damping, then steady aerodynamics, then quasi-steady aerodynamics and finally stall behavior. In this way, we gradually developed a model that evolved over time and became more and more accurate as we revised our model.

8. CONCLUSION

The aim of this project was to study several models that aim to predict the behavior of an airfoil at different airspeeds below, at and above the critical airspeed. In particular, the prediction of the critical speed and the ability of the model to accurately predict airfoil plunge and pitch displacements were studied. Through this project, we have concluded that a model implementing steady aerodynamics is able to accurately predict the critical airspeed at which flutter occurs because it is a better model of the steady-state response of the airfoil. However, a model implementing quasi-steady aerodynamics is able to more accurately predict the plunge and pitch behavior of the airfoil becuase it attempts to account for time-dependent effects of the airstream on the airfoil. Despite these conclusions, this project was limited by our ability to accurately determine some of the parameters of our model as well as account for other factors effecting the airfoil. As such, future work should take into account other non-linearities of the system as well as attempt to better determine the parameters of the system such as damping coefficients and the contribution due to dry friction.

9. FUTURE USAGE

This project would be a very suitable project for the next dynamics class. As mentioned in previous sections, we have left several key parameters out of our model, such as dry friction and a proper implementation of stall angle. These are all parts that multiple groups could take on. For instance, one group could attempt to produce a lift curve for the airfoil in the wind tunnel and compare it to available NACA0012 lift curves and study the degree to which the data from the airfoil in the windtunnel matches known NACA0012 airfoil data. Another group could attempt to implement various types of damping to determine if any one type of damping could better model the behavior of the airfoil or if it might be necessary to use a combination of different damping models (e.g. viscous and linear damping). The following are possible problem statements that could be a follow up to this project:

(1) How accurate is the airfoil in the wind tunnel relative to a standard NACA0012 airfoil?
(2) What type/types of damping will accurately model airfoil flutter?
(3) How might the damping coefficients of the airfoil flutter test stand in the wind tunnel be determined accuratelely?

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